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# DEFORMABLE TEMPLATES FOR CIRCLE RECOGNITION

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An algorithm for the circle recognition, using deformable templates, was carried out and its performance was studied. The displacement of the points from circles and the presence of noise that appear in real situations were taken into account. The deformable templates algorithm is initialized by Hough transform, which performs a rough evaluation of parameters. However due to inefficiency of the standard Hough transform, a new fast Hough transform procedure was proposed with an automatic choice of the appropriate cut on histograms and handling of splitting peaks.

Having the approximate number of arms and the corresponding initial values of the parameters from the Hough transform as input, a neural network finds circles with high resolution. To avoid getting stuck into local minima we decrease the interaction between the arms and use the simulated annealing procedure where the system is allowed to thermalize for a sequence of temperature according to the Boltzmann distribution. Besides we penalize the case in which an elastic circle stopped its evolution having not enough points on it.

Simulated data were used to study the efficiency of the algorithm depending on such factors as the noise level, displacements of the points from circles, the number of points per circle and the distance between the centre of two overlapped circles. Results show the satisfactory robustness of our algorithm to background contaminations. Then this technique was successfully applied to real data obtained in Au-Pb interactions from the RICH detectors used in CERES/NA45 experiment.

The investigation has been performed at the Laboratory of High Energies, JINR.

## Применение метода деформируемых образцов для распознавания окружностей

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Дано описание алгоритма распознавания окружностей методом деформируемых образцов и изучено поведение алгоритма в зависимости от фона и разброса измеренных точек. Для инициализации метода деформируемых образцов предварительно применяется преобразование Хафа, позволяющее грубо оценить параметры. Из-за неэффективности стандартного преобразования Хафа была предложена его быстрая модификация с автоматическим выбором порогов обрезания гистограмм и обработкой случаев сдвоенных пиков. Начальные значения параметров, полученные с помощью преобразования Хафа, использовались как входные для нейронной сети, эволюция которой позволяла найти окружности с более высоким разрешением. Чтобы избежать скатывания в локальный

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минимум энергетической функции сети, мы уменьшали взаимодействие между образцами и применили процедуру имитированного отжига, когда система «остывает», проходя последовательность температур в соответствии с распределением Больцмана. Помимо этого, были введены штрафные функции на случаи, когда на эластичной окружности при остановке ее эволюции оказывается слишком мало точек. На модельных данных была изучена эффективность алгоритма в зависимости от уровня шума, разброса точек вокруг окружности, числа точек на окружности и расстояния между центрами двух перекрывающихся окружностей. Результаты вычислений показали хорошую устойчивость нашего алгоритма к уровню шумового загрязнения. Затем эта техника была успешно применена к реальным измерениям, полученным в Au-Pb взаимодействиях на RICH-детекторе установки CERES/NA45.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

## 1. Introduction

An important problem in the area of pattern recognition is curve recognition. In the context of high energy physics, Cherenkov rings finding represents such a problem. This is a combinatorial optimization problem: given a set of detector signals, one reconstructs Cherenkov rings subject with different constraints. This is getting more important with inventing such modern detectors as, for example, RICH (Ring Imaging CHerenkov) requiring in each event the determination of parameters for tens or hundreds of rings formed by Cherenkov photons. The presence of the background noise and the appearance of several overlapping rings, along with the displacement* of the points from the circle, make inapplicable classical methods for circles fitting due to their noise sensitivity. Even the presence of one single point outlier can distort considerably circle centre or radius estimations. The same effect appears for double or triple crossing circles if you would fit them one by one because of the influence of the new circle that is formed in the crossing area.

Artificial Neural Networks (ANN) techniques, and variations thereof, have shown great power in finding approximate solutions to difficult combinatorial optimization problems [10,7,9]. The ANN technique has also been used for the tracks finding problem [11,2] and for a similar problem which is Cherenkov rings finding [4]. In [4] a non-adaptable network was used for recognizing several circles in a given pattern with encouraging results.

Deformable templates and Hough transform algorithms were used first in image processing and visual pattern recognition fields. Deformable templates approach was used in [2,3] for tracks finding problem in high energy physics. Other application of this approach can be found in [6], where the authors proposed a modification of deformable templates method, for the tracks finding problem, in case of data detected with high pressure drift tubes. Also an implementation of the elastic arms approach to tracks reconstruction, using object-oriented programming techniques, is described in [8].

An interesting method for circles fitting using RICH raw data is studied in [5]. In this article the discrete structure of the detector was taken into account, a circle is to be fit not to separate points, but to clusters of adjacent cells generated by the energy dissipation.

In our work a new deformable templates algorithm for circle reconstruction is presented and its performances are evaluated. These deformable templates are initialized by Hough

*For real data the displacement is defined as the error in determining the coordinates of the signal points.

transform. The algorithm effectively represents a merger between neuronic decision and parameter fitting that are made simultaneously by solving the equation of motion for the analogous dynamic system. In other neural networks or classical approach one then has to augment the algorithm with some fitting procedure. It would be advantageous to have an algorithm that does both the assigning and the fitting simultaneously.

The elastic arms strategy is to match the observed events (data point) to simple parameterized models (circles). An unknown subset of these data points corresponds to noise and should be unmatched.

These two methods, Hough transform and deformable templates, complement each other. The conventional Hough transform procedure performs a low resolution search for circles. It provides the approximate number of circles and a rough evaluation of centres and radii. However the existence of non-perfect circles and noise points makes the standard Hough transform less effective. Therefore a new fast Hough transform procedure is proposed. It consists of two steps: first, the coordinates of centres are found and second the corresponding radii are determined. In order to automatize the choice of the appropriate cut of histograms and to handle the problem of splitting peaks we use local averages on histograms.

The paper is organized as follows: An overview of the problem is in section 2. In section 3 we describe a modified Hough transform. Section 4 contains the deformable templates algorithm with a review of general method. Implementation issues, practical hints and final results are presented in section 5. Finally, in section 6 the reader finds the conclusions.

## 2. Overview of the Problem

When a charged particle, with a momentum $p$, passes through a medium characterized by a refraction index $n$, such that $p > c/n$, it emits light around a cone whose angle $\theta$ is given by $\cos\theta = 1/\beta n$, where $\beta = v/c$ is the particle velocity related to the speed of the light. This is called Cherenkov effect. From a front view the cone appears as a circle. As the intensity in Cherenkov light is usually low, only a few photons are emitted which has the result that the Cherenkov ring appears not like a full ring but only as several dots lying on a circle*. The aim is to reconstruct the circle that passes through these points knowing that this one can be deformed as the light originating from the particle usually goes through different materials and therefore can be diffracted or even reflected. Furthermore, one has to deal with some noise inherent to any low sensitivity detection.

In this article we reconstruct circles with radii in the $[R_{in}, R_{max}]$ range, knowing the displacement (in case of Monte Carlo circles the input signal points are smeared adding a randomly uniformly distributed value between zero and a maximum named displacement) of the points from the circles and the required number of points per circle.

*The problem is considered here in a simplified view by omitting the influence of a detecting system, which provides in fact a lot of small data distorsions and random displacement from the circle. This influence is overcome on a preprocessing stage, although could be included in data processing explicitly like in [5].

## 3. Hough Transform

The idea of the elastic arms algorithm is to match the observed events into a known parameterized model. To ensure a fast convergence towards a high quality solutions avoiding the local minima, the elastic arms algorithm must be initialized with approximate values for the positions of the centres and the radii of the circles. Then, as a first step we need a method that provides us the approximate number of circles and the approximate values for their centres and radii. The Hough transform is appropriate for this task.

The use of the Radon transform, or in its discrete form, the Hough transform, is not new in particle physics. A Radon transform of a density $\rho(\mathbf{r})$ is defined by line integrals over a special class of curves:

$$R(\mathbf{r}, \mathbf{P}) = \int \mathbf{d}\tau\rho(\mathbf{r}_{\mathbf{p}}(\tau) + \mathbf{r}), \tag{1}$$

where $\mathbf{r}_{\mathbf{p}}(\tau)$ is the curve specified by parameters $\mathbf{p}$ such that $\mathbf{r}_{\mathbf{p}}(0) = 0$, and $\mathbf{r}$ is a point on the curve. If the phase space is discretized, Radon transform converts into the Hough transform. Radon transform is intrinsic instable [12].

An event is defined as a set $\mathbf{x} = \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N_p}$ of $N_p$ signal points. These $\mathbf{x}_i$'s are bidimensional. Each event also corresponds to a set of circles (Cherenkov rings) and the aim of the algorithm is to find these circles knowing $\mathbf{x}$.

If we denote the coordinates of three arbitrary nonlinear points by $(x_i, y_i)$ $i = 1, 2, 3$, the circle which passes through these points has for centre the $(x_0, y_0)$ coordinates

$$x_0 = \frac{1}{2}\,\frac{(x_2^2 - x_3^2 + y_2^2 - y_3^2)(y_1 - y_2) - (x_1^2 - x_2^2 + y_1^2 - y_2^2)(y_2 - y_3)}{(x_2 - x_3)(y_1 - y_2) - (x_1 - x_2)(y_2 - y_3)}, \tag{2}$$

$$y_0 = \frac{1}{2}\,\frac{(x_1^2 - x_2^2 + y_1^2 - y_2^2)(x_2 - x_3) - (x_2^2 - x_3^2 + y_2^2 - y_3^2)(x_1 - x_2)}{(x_2 - x_3)(y_1 - y_2) - (x_1 - x_2)(y_2 - y_3)}, \tag{3}$$

and for radius:

$$R = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}, \quad i = 1, 2, 3. \tag{4}$$

Given a set of points, one can construct all the possible circles. This will give a set of centres and radii. All these circles are drawn in the space of parameters. This space is then discretized and the entries are histogramed, one divides the $(x_c, y_c, R_c)$ space up into boxes and counts the number of circles in each box. If the towers in the histogram exceed some thresholds, then the corresponding parameter values define a potential circle.

As we said before, one makes a histogram in parameters space to find the most popular parameter within the resolution. Because of the granularity (the dimension of the boxes on the histogram) of the histogram and the precision of the calculus, even for a Monte Carlo generated circle in the absence of the noise and without any displacement of the points from the circle, we shall have many solutions for the parameters $(x_c, y_c, R_c)$. These facts together with the existence of non-perfect circles and the noise makes the standard Hough transform not so effective.

We therefore use a Hough transform which consists in two steps: first the centres are determined and after that the radii are calculated using one by one the determined positions for the centres.

There are some parameters that determine the quality of the procedure. Choosing these parameters we must keep in mind all the time that:

- The deformable templates need in order to be initialized only rough solutions, we don't need a very good precision as in the case of other methods that gives candidates for fitting.
- It is easy to observe (see Fig.3) that for the elastic arms algorithm it is preferable to deal with the situations in which the initializing set has some extra circles than it would be initialized with a number of circles less than necessary, when it will lose circles.

As a first step, using formulae (2), (3), we calculate the centres of the circles described by each possible set of three points that accomplishes some geometrical cut:

- The distances between each pair of points must be smaller than the diameter of the biggest possible circle defined by $R_{max}$.
- The condition of nonlinearity of these three points.

In the space of the centres, we made a two-dimensional histogram ($x_c$, $y_c$, $N$), where $x_c$ and $y_c$ represent the coordinates of the centres; and $N$, the number of possible combinations in each box. Such kind of histogram is shown in Fig.1.

The main problem is to extract the information from the histogram, i.e., to choose a level that defined «the most popular parameters». It is impossible to choose only one criterion for all the possible cases, then we used three concurrent types of cuts:

1. the number of possible combinations divided by three $C^3_{RNPP}/3$, where *RNPP* represents the required number of points on a circle;
2. the average of $N$ over the histogram;
3. number three that represents the minimal number of points that can describe only one circle.

The level of cut will be the maximum between these three numbers.

As we observed in practice, if the level of the noise and (or) the number of circles together (separately) grow

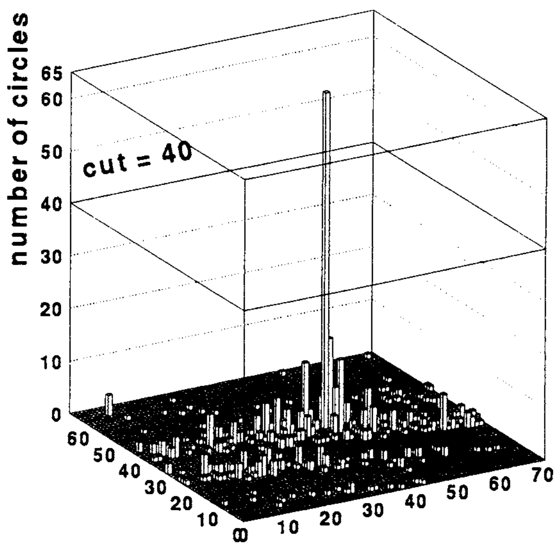


Fig.1. Two dimensional histogram of centres ($x_c$, $y_c$, $N$). One Monte Carlo circle with 10 points on it, noise/signal ratio is 100%, displacement of points from the circle is 0.1 and the level of cut is 40

up, the average over the histogram gives the cut. In the case of a small number of circles and (or) a low level for noise the number of combinations dominates. If we have a large displacement from the real circles and a small number of signal points, the number three gives the cut.

In the second step, for each found centre we define a ring centered in the $(x_c, y_c)$ and limited by $R_{min}$, $R_{max}$. For each data point situated inside this ring we use eq.(4) to calculate the polar radius related to this centre. Then, for each centre we made a histogram of these radii. The histogram cut is chosen as the maximum between: the required minimal number of points on circle divided by two, the average over the histogram and the number three. All the time the granularity for the radius is chosen a little bit bigger than the granularity for centres because of the ambiguities in determining the position of the centres. If we obtain for one centre two possible radii or more we keep only this particular one that corresponds to the highest tower on the radii histogram. At the first sight it seems the lower granularity would improve the precision of our result. However it is not true. The granularity of the histograms must be chosen very carefully. Some problems appear both at smaller granularity and at larger one. If the granularity is too small, the computational requirements grow very rapidly and some splittings of the peaks appear that results in loosing «good» circles. We shall have in the same time false circles because the average of the $N$ over the histograms will decrease that means false solutions, wasting of computer resources. If the granularity is too large,it will be possible to put together the combination corresponding to two or more than two circles that results again in loosing of «good» circles due to two mechanisms: the overlap of the solutions and the increase of the level of cuts in both types of histogram. We must keep in mind in the same time that if we want to find circles for which the displacement of the points is two times larger than the granularity, we must increase the granularity.

In order to have better solutions (even in the case of splitting peaks or if the displacement of the points from the circle is big) we give as good value for centres and radii, not the centre of the square (or the segment) defined by granularity, but a weighted average over the neighbouring cells.

This algorithm was implemented as a Fortran-77 subroutine in a main program.

## 4. Deformable Templates (Elastic Arms) Algorithm

The deformable templates algorithm is initialized by Hough transform. Having the approximate number of arms and the corresponding initial values of the parameters from the Hough transform as input, a neural network finds circles with a higher resolution.

*4.1. Review of the Algorithm.* As we said before, an event is defined as a set $\mathbf{x} = \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{N_p}$ of $N_p$ signal points, where $\mathbf{x}_k$ vectors are bidimensional. Each event also corresponds to a set of rings (circles). In order to find these rings (circles), a set $\boldsymbol{\pi} = \{\boldsymbol{\pi}_1, .., \boldsymbol{\pi}_M\}$ of $M$ deformable templates, or elastic arms, is introduced, where an arm $a$ is completely described by its $P$ parameters: $\boldsymbol{\pi}_a = (\pi_a^1, \ldots, \pi_a^P)$.

We reduce the circle finding problem to finding minimum of the following energy function:

$$E(\{S_{ia}\};\boldsymbol{\pi}) = \sum_{i=1}^{N} \sum_{a=1}^{M} S_{ia} M_{ia}(\mathbf{x},\boldsymbol{\pi}) + \lambda \sum_{i=1}^{N} (\sum_{a=1}^{M} S_{ia} - 1)^2, \tag{5}$$

where $S_{ia}$ is a binary decision unit (neuron) defined as:

$$S_{ia} = \begin{cases} 1 & \text{if signal } i \text{ is assigned to arm } a \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

$M_{ia}(\mathbf{x},\boldsymbol{\pi})$ is the Euclidean distance between signal point $i$ and arm $a$. Since each signal can belong only to one circle or to no circle at all, $E$ must be minimized with the condition:

$$\sum_{a=1}^{M} S_{ia} = 1 \text{ or } 0 \quad \text{for each } i. \tag{7}$$

The parameter $\lambda$ imposes a penalty if $\sum_a S_{ia} = 0$, that means, signal $i$ is not assigned to any arm. In this way the parameter $\lambda$ governs the amount of noise the algorithm allows for.

In order to avoid local minima when minimizing $E$, one often introduces thermal noise into neural system. A commonly used procedure for doing this is simulated annealing where the neural system is allowed to be thermalized for a sequence of temperatures $T_n > T_{n-1} > \ldots > T_0$ according to the Boltzmann distribution [2], [1]:

$$P(\{S_{ia}\};\boldsymbol{\pi}) = \frac{1}{Z} e^{-\beta E(\{S_{ia}\};\boldsymbol{\pi})}, \tag{8}$$

where $\beta = 1/T$ and $Z$ is the partition function.

We proceed by calculating the marginal probability distribution:

$$P_M(\boldsymbol{\pi}) = \sum_{\{S_{ia}\}} P(\{S_{ia}\};\boldsymbol{\pi}) \tag{9}$$

by summing out the neuronic degrees of freedom, $S_{ia}$. We obtain:

$$P_M(\boldsymbol{\pi}) = \frac{1}{Z} e^{-\beta E_{\text{eff}}(\boldsymbol{\pi})}, \tag{10}$$

where the effective energy is defined as:

$$E_{\text{eff}}(\boldsymbol{\pi}) = -\frac{1}{\beta} \sum_i \log\left( e^{-\beta\lambda} + \sum_a e^{-\beta M_{ia}} \right). \tag{11}$$

The most probable configuration according to eq.(10) corresponds to the minimum of $E_{\text{eff}}$. Using a gradient descendent method to minimize $E_{\text{eff}}$ one gets an updating rule:

$$\Delta\pi_a^{(k)} = -\eta_a^{(k)} \frac{\partial E_{\text{eff}}}{\partial \pi_a^{(k)}} = -\eta_a^{(k)} \sum_i \hat{V}_{ia} \frac{\partial M_{ia}}{\partial \pi_a^{(k)}}, \tag{12}$$

where $\eta_a^{(k)}$ is the updating parameter and the Potts factor $\hat{V}_{ia}$ is given by:

$$\hat{V}_{ia} = \frac{e^{-\beta M_{ia}}}{e^{-\beta\lambda} + \sum_{b=1}^{M} e^{-\beta M_{ib}}}. \tag{13}$$

*4.2. Circles as Elastic Arms.* In our case the templates are circles with variable centre and radius. The template is defined in this case by the coordinates of centre and by the radius $(x_a, y_a, R_a)$.

A circle $(x_a, y_a, R_a)$ is given by the parametric equations:

$$x(t) = x_a + R_a \cos(t), \quad y(t) = y_a + R_a \sin(t), \quad t \in [0, 2\pi). \tag{14}$$

The Euclidean distance $M_{ia}$ between a circle $(x_a, y_a, R_a)$ $a = 1,\ldots, M$ and a signal point $(x_i, y_i)$ $i = 1,\ldots, N$ is given by:

$$M_{ia}(x_a, y_a, R_a; x_i, y_i) = \left| \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2} - R_a \right|. \tag{15}$$

Using the energy function for elastic circles $(x_a, y_a, R_a)$ as eq.(5) and the effective energy as eq.(11) we obtain the updating rule for circles:

$$\begin{aligned} \Delta x_a &= -\eta_a \frac{\partial E_{\text{eff}}}{\partial x_a} = -\eta_a \sum_i \hat{V}_{ia} \frac{\partial M_{ia}}{\partial x_a}, \\ \Delta y_a &= -\eta_a \frac{\partial E_{\text{eff}}}{\partial y_a} = -\eta_a \sum_i \hat{V}_{ia} \frac{\partial M_{ia}}{\partial y_a}, \qquad a = \{1,\ldots M\}, \\ \Delta R_a &= -\eta_a^r \frac{\partial E_{\text{eff}}}{\partial R_a} = -\eta_a^r \sum_i \hat{V}_{ia} \frac{\partial M_{ia}}{\partial R_a}, \end{aligned} \tag{16}$$

where $\eta_a$ is the updating parameter for the coordinates; and $\eta_a^r$, for the radius. $\hat{V}_{ia}$ is the Potts factor (eq.(13)).

The derivatives of $M_{ia}$ with respect to the parameters are:

$$\frac{\partial M_{ia}}{\partial x_a} = \frac{x_a - x_i}{\sqrt{(x_a - x_i)^2 + (y_a - y_i)^2}} \operatorname{sign}\left(\sqrt{(x_a - x_i)^2 + (y_a - y_i)^2} - R_a\right), \tag{17}$$

$$\frac{\partial M_{ia}}{\partial y_a} = \frac{y_a - y_i}{\sqrt{(x_a - x_i)^2 + (y_a - y_i)^2}} \operatorname{sign}\left(\sqrt{(x_a - x_i)^2 + (y_a - y_i)^2} - R_a\right), \tag{18}$$

$$\frac{\partial M_{ia}}{\partial R_a} = \operatorname{sign}\left(\sqrt{(x_a - x_i)^2 + (y_a - y_i)^2} - R_a\right). \tag{19}$$

*4.3. The Behavior of the Algorithm.* The templates have Gaussian distributions centered around the arm values with width of Gaussians depending on the temperature. When the temperature is high, each arm can attract many signals, the arm having a large sensitive area. The relative importance of the different signals is measured by the Potts factor (eq.(13)). Also the Potts factor implicitly contains a repulsive force between each two arms. This interaction between arms is generated by the sum $\sum_{b=1}^{M} e^{-\beta M_{ib}}$. Considering only two circles $a$ and $b$ with $x_a > x_b$, the interaction force between them has the expression:

$$\mathbf{F}_{ab} = -\nabla E_{\text{eff}} = \frac{\partial E_{\text{eff}}}{\partial(x_a - x_b)}\,\mathbf{i} + \frac{\partial E_{\text{eff}}}{\partial(y_a - y_b)}\,\mathbf{j}. \tag{20}$$

Its projection on $Ox$ axis is:

$$\frac{\partial E_{\text{eff}}}{\partial(x_a - x_b)} = \sum_i \sum_{c=a,b}^{M} \hat{V}_{ic} \frac{\partial M_{ic}}{\partial(x_a - x_b)} \tag{21}$$

and with the following geometrical relation between $M_{ic}$ and $x_a - x_b$:

$$x_a - x_b = \sqrt{(R_a + M_{ia})^2 + (R_b + M_{ib})^2} \tag{22}$$

finally we have:

$$(F_{ab})_x = \frac{\partial E_{\text{eff}}}{\partial(x_a - x_b)} = \sum_i \sum_{c=a,b}^{M} \left( \hat{V}_{ic} \frac{x_a - x_b}{R_c + M_{ic}} \right) > 0. \tag{23}$$

In eq.(23) $(x_a - x_b) > 0$ and $\hat{V}_{ic}$ is the Potts factor (eq.(13)) always greater than zero.

Analogous for $Oy$ axis:

$$(F_{ab})_y = \frac{\partial E_{\text{eff}}}{\partial(y_a - y_b)} = \sum_i \sum_{c=a,b}^{M} \left( \hat{V}_{ic} \frac{y_a - y_b}{R_c + M_{ic}} \right) > 0. \tag{24}$$

This repulsive force is a winner-takes-all structure. If the coordinates of the centres are the same ($x_a = x_b$ and $y_a = y_b$), the repulsive force vanishes ($F_{ab} = 0$).

The repulsive force, generated by the $V_{ia}$ term, is useful when the main problem is to separate two overlapping circles (Fig.2) but it is also an important source of local minima. Many local minima appear when the sum $\sum_{b=1}^{M} e^{-\beta M_{ib}}$ is not $e^{-\beta M_{ia}}$. To avoid this, we use a new simplified form of Potts factor without interaction between circles:

$$\hat{V}_{ia} = \frac{e^{-\beta M_{ia}}}{e^{-\beta\lambda} + e^{-\beta M_{ia}}}. \tag{25}$$

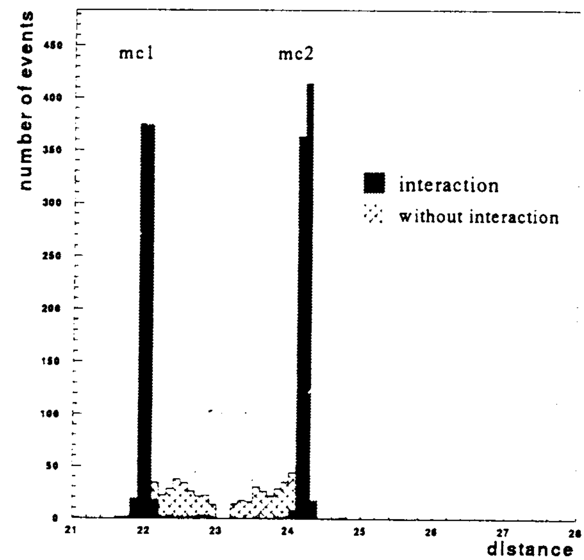



Fig.2. The influence of the repulsive force. One histogram uses Potts factor, eq.19 (with interaction); and other, eq.25 (without interaction)

The λ parameter governs the relative importance of signals that are not associated with a circle. If we have noise we must increase λ according to the noise level.

To change dynamically the update rates and to know and penalize the situation when a circle is in a local minimum we define a new parameter associated with a circle: the number of points per circle at one moment in time (*NPP*).

We say that a circle is captured by a local minimum when it stays long time (few iterations) in the same position in the parameters space ($x_c$, $y_c$, $R_c$) having only one, two or three points on it.

For such cases, we increase the updating rates and move randomly this particular circle in another place of the space of parameters. It is like increasing the temperature only for that circle.

If *NPP* is equal or more than the required minimal number of points on a circle, the circle is a «good» one and we decrease the update rates for it. When *NPP* is zero the update rates increase at initial values. Thus update rate is defined as:

$$\text{updat[a]} = \begin{cases} \text{Minimum updat if } NPP > \text{Required } NPP \\ \text{Maximum updat if } NPP = 0 \\ \text{Medium updat other else} \end{cases} . \tag{26}$$

If the algorithm is initiated by too many arms by the Hough transform, the extra circles can be:

1. attracted to noise;
2. attracted to points belonging to Cherenkov rings;
3. attracted to a circle upon which another arm has already settled on.

*4.4. Computer Simulation.* We implemented successfully the deformable templates algorithm as a *C* subroutine of a main program. Translation of the *C* code to *C++* code is easy because the *C* structures defined can be redefined as classes and the functions as member functions or friend functions of these classes (see [8]). In this subsection we give a set of prescriptions and hints of how to ensure a good and rapid convergence in a way that is as problem independent as possible, how to avoid getting stuck and how to go out from local minima.

There are many parameters that must be properly set up. A neural networks are very sensitive to these parameters. One of them is the temperature. A nonlinear phenomenon like phase transition which appears at a specific temperature determines different behavior of the neural system for different temperatures.

At the starting temperature $T_H$, a set of template arms is placed according to the Hough transform values for the parameters $(x_a, y_a, R_a)$. As the temperature is lowered the different arms are attracted, with different forces, more intensely by the nearby signals. Next step is to minimize effective energy $E_{\text{eff}}(\{x_a, y_a, R_a\})$ (eq.(11)) using the gradient descent equations (eq.(16)).

As we mentioned earlier the width of Gaussian around each arm depends on the temperature, therefore $T_H$ and $T_F$ should be chosen with respect to magnitude of the dynamic range. If the high temperature $T_H$ is too high, it can destroy the initial Hough configuration. If it is too low the simulated annealing will not work properly. If the final temperature $T_F$ is not low enough, the level of neural networks noise would be too high and can destroy a good configuration.

In eq.(12), the partial derivatives $\partial E_{\text{eff}}/\partial x_a$, $\partial E_{\text{eff}}/\partial y_a$, $\partial E_{\text{eff}}/\partial R_a$ have different magnitudes. We must therefore use one update rate for radius and another one for coordinates of the centres. It is useful to have a dynamic change of updating rates for each circle because one circle can have, in one moment during its evolution, zero points on it and another one can have a lot of points on it. Each circle has its own updating rates. When initiating the update rates we want a smooth transition of the parameters from Hough parameter values again; if the initial values of the updates are too big, we destroy the Hough configuration; if they are too small, the configuration of circles will not change enough.

The magnitude of the partial derivatives depends strongly on the magnitude of our signal coordinates $(x_i, y_i)$. In order to make the update rates less dependent on different applications we rescale all signals to some predefined dynamic range. Because of that we shall give our result in some conventional units.

The gradient descent method is only one way of minimizing $E_{\text{eff}}$. A very simple way to improve the gradient descent method is to introduce the so-called momentum term. Each degree of freedom $\pi_a$ is given some inertia of momentum. In other words $\Delta\pi_a(t)$ get a contribution from $\Delta\pi_a(t-1)$ according to [2]:

$$\Delta\pi_a(t) = -\eta\nabla E_{\text{eff}} + \alpha\Delta\pi_a(t-1). \tag{27}$$

This means that $\pi_a$ feels an average downhill «force» when moving on the energy surface. We introduce momentum term to prevent the energy from oscillating and to make the minimizing more effective. The $\alpha$ parameter is taken as 0.5.

Deformable templates algorithm checks whether the radius of elastic circles has values in the input range $[R_{min}, R_{max}]$, the centres are on the detector area and all circles have at least the number of points per circle equal to the required minimal number of points. There are other characteristics of the output circles that are not checked, for this reason we introduced some final cuts. The elastic arms algorithm does not check if there are very close circles, which can be considered as the same circle. Therefore, if we obtain two or more circles with very close centres and approximative equal radii, we keep as good circle only one of them.

When we work with large displacement of points from circle we observe some situations in which a pair of circles situated in the same region have many points in common. If one circle has all its points included among the points that defined other circle, we shall keep as a good, the only one of circles with the biggest number of points on it.

For real data, clusters of signals appear many times on the detector then we obtain, as output of deformable templates, some circles with a non-uniform angular distribution of points on the circle. That kind of circles must also be eliminated by final cuts.

*Summary of the Method.* A summary of elastic arms algorithm is presented below:

1. Obtain an initial set of arms $(x_a, y_a, R_a)$ $a = 1, N_{Hough}$ from Hough transform.

2. Rescale the signals $(x_i, y_i)$ and arms $(x_a, y_a, R_a)$ to the dynamical range.

3. Choose the initial update rates according to the dynamic range.

4. Choose values of $\lambda$, $T_H$, $T_F$.

5. For a sequence of temperatures [2] $T_n = kT_{n-1}$, $k = 0.95$, update according to eqs.(16) and:

5.1. Check the *NPP* and change update rates according to the rules (eq.(26)).

6. Make it converge at $T = T_F$ until $F_{eff}$ is not changing, by lowering the update rates [2]:

$$\eta_c = \varepsilon\eta_c, \quad \eta_r = \varepsilon\eta_r, \quad \varepsilon = 0.9.$$

At each loop:

6.1. Check the *NPP* and change update rates according to the rules (eq.(26)).

6.2. Check for local minima.

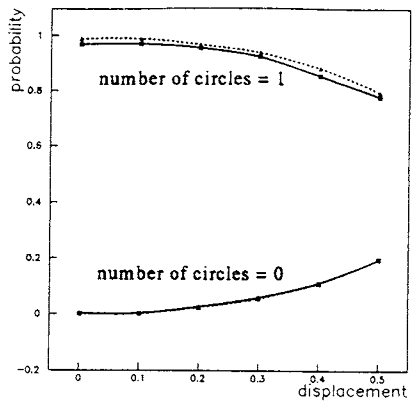
7. Make cuts.

## 5. Simulation and Results

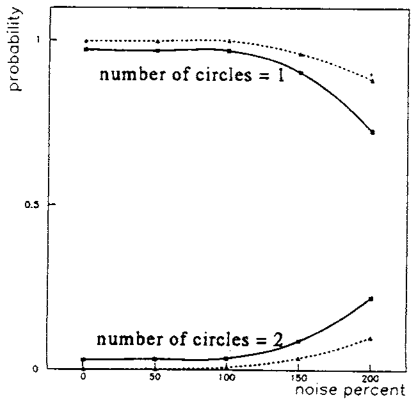
We study in case of one Monte Carlo circle as input, the output of algorithm using different values for displacement of points from circle, uniform distributed noise, fixed number of points per circle

To study the influence of the displacement of points from circle, 1000 events were generated with one Monte Carlo circle, having ten points per circle and radius 10, using a noise/signal ratio 100%. The detector area in which the circle was generated was $42 \times 42$

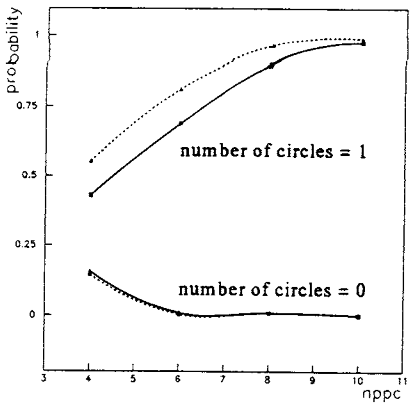
Fig.3. Probability to obtain zero, one or two circles after Hough (line) and after deformable templates (dash-line). Influence of the displacement (a), of the noise/signal ratio (b) and of the number of points per circle (c)




a



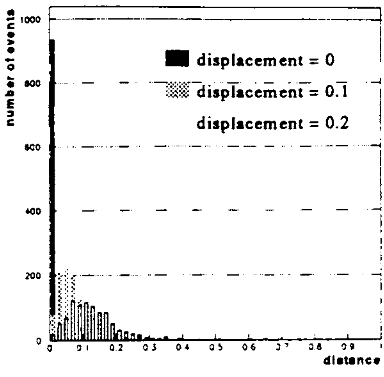

b



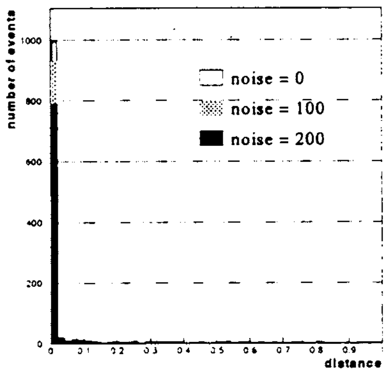

c

(in the same units). The probability to obtain no circle is represented in Fig.3a (bottom) for different displacements in the range [0, 0.5] (in the same conventional units). As we can see there is not a strong affect of the displacement on the number of circles obtained as output in the range [0, 0.3]. The probability to lose a good circle grows up slowly with the displacement. The dash line corresponds to the output of deformable templates; and the solid line, to the results given by Hough transform. In Fig.3a (up), the probability to find one circle, that means a right result, is represented. We observed that the probability to obtain a good result is very high and decreases slowly when the displacement grows up. We obtained «good» results in 80% of cases for 0.5 displacement.

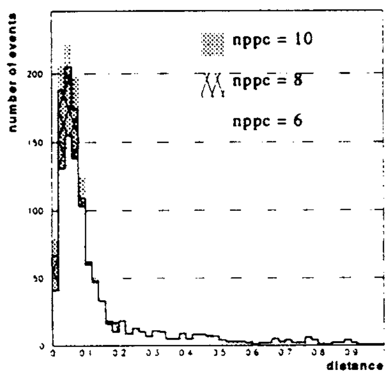
In Fig.3b it is shown in which way the variation of the noise/signal ratio modifies the probability to find one circle (up), respectively two circles (bottom). We used Monte Carlo circles generated under the same conditions, 0.1 displacement and the noise/signal ratio in the range [0, 200%]. It is easy to observe that: the probability to find only one circle decreases when the noise/signal ratio increases because noise points can form other circles, then the probability to find more than one circle increases. Deformable templates are less sensitive to noise than Hough transform. Many times, when Hough transform output contains one or very seldom two supplementary circles (for one Monte Carlo circle as input), deformable templates will have only one output circle, the second circle will be the same as the first after convergence or disappears. This explains why the Hough transform curve is below in Fig.3b (up) and above in Fig.3b (bottom). We can observe that the increase of the noise/signal ratio results in the appearance of false solutions. The probability to obtain false solutions is small, 5%, at 100% noise/signal ratio.




a




b




c

Fig.4. Histogram of the distance between the centre of Monte Carlo generated circle and the obtained circle* for 0.01 and 0.2 displacement, 100% noise, 10 points per circle (a); for 0%, 100% and 200% noise, 0.1 displacement, 10 points per circle (b) and for 6, 8 and 10 points per circle. (0.1 displacement, 100% noise) (c)

*As we have shown, in case of noise data we obtain sometimes 2 output circles (less than 1% of cases for 100% noise/signal ratio, 0.1 displacement and 10 points per circle) (Fig.3b). In this case we put into the histogram the distance between the MC circle and the closest output circle [13]

The probability of obtaining good solutions is very sensitive to the number of points per circle (Fig.3c). Under the same conditions we generate Monte Carlo circles described by ten, eight, six and four points. We can observe that, when the number of points per circle decreases, the probability to lose solutions grows up, being approximately 20% if we want to reconstruct circles described by no more than four points. The probability to obtain good solutions is in the same case 60% that means that in case of four points on circle sometimes false solutions appear. This probability grows up very rapidly with the number of points per circle, being 80% for six points per circle and almost 100% for eigth, respective ten points per circle. The probability to lose the solution is almost zero for six, eight, ten points per circle. The situation in which we lose solutions is more disadvantageous than the situation in which we obtain false solutions because, in the latter case, the false solutions can be eliminated by the final cuts.

We studied the precision of the method using a statistics of the absolute values of the distances between the centre of Monte Carlo circle and the centre of the corresponding output circle given by deformable templates.

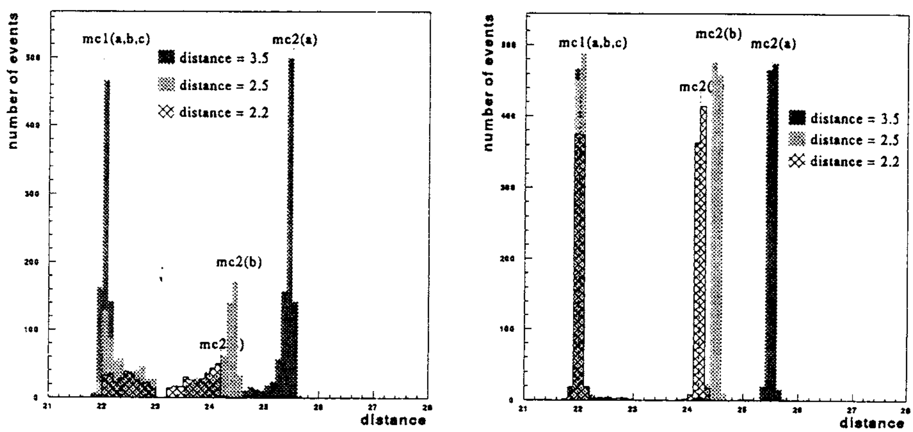
We study how the precision depends on variations of displacement, noise/signal ratio, required minimal number of points per circle. We observe that the influence of the displacement is very important. Very small variations in the displacement value result in important modifications of the shape of the histograms. In the absence of the displacement in more than 95% of the cases the distance is less than 0.01. This distance grows up very rapidly with the displacement being in the same percent of cases twenty times bigger in the case of 0.2 displacement. If the displacement increases from 0 to 0.2 the information carried by useful points decreases. For the 0.2 displacement the error in measuring centre has the same value 0.2. Almost all histograms of distance will spread into [0, 0.2] range (Fig.4a). The influence of the noise is shown in Fig.4b. As we can see the noise influence is not important in this situation. Even for 200% noise/signal ratio in more than 80% of cases the distance is less than 0.01. Then we can observe that the algorithm is very robust to noise.

We studied the influence of number of points per circle in the range 6 ÷ 10 at 0.1 displacement and 100% noise/signal ratio. As we could see, the influence of the number of points per circle is more important than the influence of the noise/signal ratio. The precision of results decreases with the number of points per circle because the amount of information about the circle decreases.

The histograms in this case depend strongly on displacement. In Fig.4c we use 0.1 displacement and different number of points per circle. If the displacement increases the required *NPP* must be increased to maintain the same amount of information about the Monte Carlo circle.

Deformable template approach performs a simultaneous fit of circles. When two circles are present, from one circle point of view, the points corresponding to other circle are noise points. If two circles are very close and have the same radius, the relative disturbing effect increases.

The histograms of $x$ coordinates of centres for two crossing circles, without interaction (given by Potts factor (eq.(25)), are shown in Fig.5a corresponding to three various distances between centres. The most difficult case when circles have the same radius was chosen. Each elastic arm is attracted by both sets of points. The attraction force increases




a b

Fig.5. Histograms of $x$ coordinates of centres for two crossing circles at different distances without interaction (Potts factor, eq.25) (a) and with interaction (Potts factor, eq.19) (b)

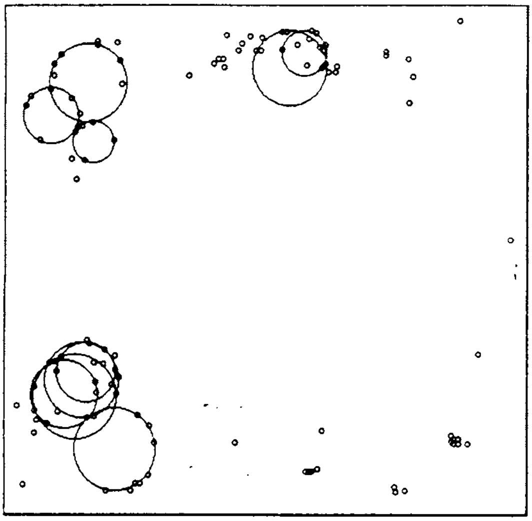


Fig.6. Output of the algorithm (RICH 2, detectors from CERES NA45 experiment)

when the distance decreases. The effective energy for only one elastic circle creates an asymmetrical shape of the histograms. Also when the distance decreases the distribution of centres becomes wider and the probability to have a small distance between output centres decreases as we can see at distance 2.2.

Using Potts factor (eq.(13)) the circles will interact, and as we demonstrated in subsection 4.3 a repulsive force will change the behaviour, of two crossing circles. Figure 5b shows a histogram represented under the same conditions as in Fig.5a, but we use eq.(13) instead of eq.(25) to compute the Potts factor. As one can see the repulsive force will generate the decrease of the attractive force of points belonging to other close circle.

*5.1. Real Data.* This technique was applied to real data obtained in lead-gold interactions from RICH detectors used in CERES/NA45 experiment. The RICH detectors are principally used as hypothesis testing devices to identify a particle with a known momentum using $\theta$ and a number of photons emitted by the particle.

In our example we identify pions with $p_t > 4.5$ GeV (asymptotic radius = 16.2 pads) in RICH 2. In Fig.6, all results of the ring reconstruction that can be given by pions (we use $R_{min} = 5$, $R_{max} = 17$, displacement = 0.2) and number of points per circle $\geq 6$ are shown.

Looking at the results we may say that the deformable templates algorithm is useful in identification of rings given a very good simultaneous fitting with a satisfactory efficiency. During data analysis we observed that the choice of parameters (required *NNP*, displacement, range of radii) in accordance with the experimental set-up is very important for the algorithm ability to recognize the Cherenkov rings.

## 6. Conclusions

Pattern recognition has always been an important part of high energy physics data analysis. We propose a circle finding method that combines the matching and the fitting problem into a single algorithm. It goes from coarse to fine resolution by using a Hough transform to initialize a set of elastic arms. A simultaneous fitting of these input arms is performed by deformable templates method finding the circles with high precision. The elastic arms approach is very similar to human processing for this kind of recognition problem. A human being looks for circles in a global way and then makes fine-tuning adjustments. Our approach gives rise to high quality solutions. It is straighforward to be implemented on a parallel processor.

Our use of Potts factor as a repulsive force is similar to using robust weights [5]. Therefore the algorithm is closely related to robust statistics, it ignores noise data to a required level. The elastic arms approach has the advantage of being flexible to host a variety of experimental set-ups. With elastic templates method the templates have a fuzzy edges, which play a very important role in the dynamics by smoothing over local minima. At first, when temperature is high, those edges should be wide and then adiabatically reduced as the template homes in a true minimum.

The method could be generalized to arbitrary dimensions with arbitrary templates, as long as those templates can be parameterized. Elastic templates method is well suited to complex pattern recognition tasks where a priori knowledge constrains strongly the possible classes of patterns. Thus for complex high energy problems all the information known a priori must be used to extract the pattern from the data.

On a practical side, elastic templates dynamics offers the advantage of being able to deal directly with continuous data distributions and to perform automatically an adaptive nonlinear fit to that data.

Concluding, we can say that our deformable templates algorithm is a considerable contribution to pion physics since it can be applied in cases of arbitrary ring radii where known conventional algorithms are helpless.

## References


1. Ohlsson M. — Comput. Phys. Commun., 1993, v.77, p.19.
2. Ohlsson M., Peterson C., Yuille A. — Comput. Phys. Commun., 1992, v.71, p.77.
3. Gyulassy M., Harlander H. — Comput. Phys. Commun., 1991, v.66, p.31.
4. Altherr T., Seixas J. — CERN-TH-6133/91, July, 1991.
5. Chernov N., Kolganova E., Ososkov G. — JINR Communication E10-95-468, Dubna, 1995.
6. Baginyan S., Baranov S., Glazov A., Ososkov G. — JINR Communication E10-94-328, Dubna, 1994.
7. Peterson C., Rögnvaldsson T. — CERN School of Computing. 1991.
8. Blankenbecler R. — Comput. Phys. Commun., 1994, v.81, p.318.
9. Denby B. — Comput. Phys. Commun., 1988, v.49, p.429.
10. Stimpfl-Abele G., Yepes P. — Comput. Phys. Commun., 1993, v.78, p.1.
11. Peterson C. — Nuclear Instruments and Methods in Physics Research, 1989, v.A279, p.537.

12. Blom J., Th. C., De Laat A.M., Kuijer P.G. — Proceedings of the Third International Workshop on Software Engineering, Artificial Intelligence and Expert Systems for High Energy and Nuclear Physics, Oberammergau, Germany, Oct. 4–8, 1993.
13. Adams D.L., Eppley G., Skeens J. — SDC-91-131, Nov. 11, 1991.

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